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"**Compounded TONA**" is recommended as the 2nd alternative interest rate benchmark to Yen LIBOR for "Lending" and "Bonds", and is currently used as an alternative interest rate benchmark to Yen LIBOR for "Derivatives" in compliance with ISDA Master. However, in general, "Compounded TONA" is thought of as something very difficult to understand. The primary reason for this seems to be that "TONA is not to be used by itself, but as a reference interest rate calculated by compounding. In this report, we would like to introduce the calculation method[1] of Compounded TONA so that you can feel more familiar with Compounded TONA.

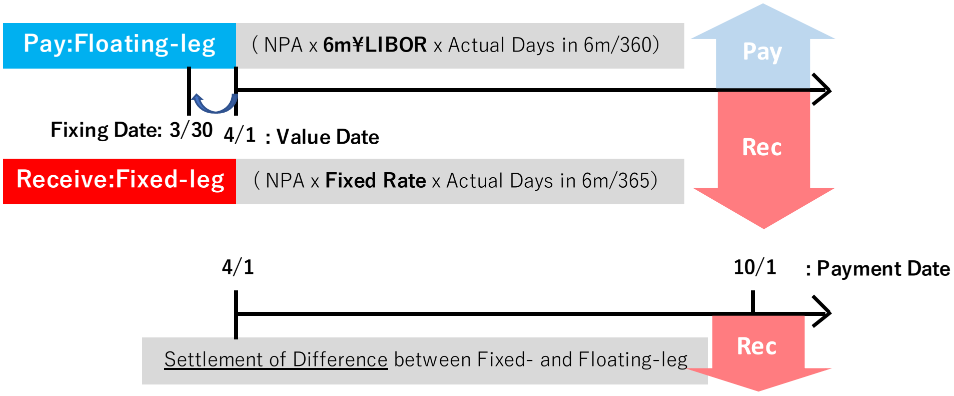
1. LIBOR Swap

IRS (: Interest Rate Swap) is one of the derivatives, and the most common transaction is to exchange fixed and floating interest rates in the same currency. Until now, the most common transaction in Tokyo Yen IRS markets has been the exchange of 6-month fixed-rate interest and 6-month Yen LIBOR (floating-rate) interest in line with the interest payment cycle of Japanese government bonds (twice a year every 6 months).Since IRS is an off-balance sheet product, no principal is exchanged. The principal is called "**Notional Principal Amount** (: **NPA**)" because it is used only for interest calculation. The only thing actually exchanged is the difference between the fixed-rate interest and the floating-rate interest (" **Settlement of Difference**").

The figure below shows the cash flow (Downward: Receiving, Upward: Paying, Horizontal Axis: Flow of Time) of a standard Yen LIBOR swap (: Plain Vanilla) transaction for receiving 6-month fixed-rate

[1] Reference to: “MUTAN ON AVERAGE (: Definitions of overnight average rates” (“**TONAW**” in REFENITIVE)

interest against paying 6-month floating-rate interest.



LIBOR is "**Setting in Advance**", which means that LIBOR is to be fixed 2 business days (3/30: **Fixing Date**) prior to the start of the interest calculation period (4/1: **Value Date**). Interest is calculated separately for Fixed-leg and Floating-leg, and only the difference between Fixed-leg and Floating-leg is settled at the end of the interest calculation period (10/1: Payment Date).

2. TONA Swap and Yen OIS

"**TONA swap**" is an off-balance sheet product that exchanges a fixed interest rate and Compounded TONA (“**Setting in Arrears**”) for a fixed period of time. Regarding TONA swap longer than 1 year, the interest is usually[2] exchanged once a year for 12 months of fixed-rate interest and Compounded TONA interest. In the case of 3-month TONA swap, for example, the fixed-rate interest and Compounded TONA interest for 3 months are exchanged only once.

TONA swap, which uses Compounded TONA as its reference interest rate, was started to trade in 1997 as a financial instrument to reflect the monetary policy of BOJ and was called "**Yen OIS**" (: Overnight Index Swap) at that time. It was a short-term financial instrument whose transaction period was limited to a maximum of 1-2 years. During the phase of BOJ's rate hike in 2006-2007, it attracted attention as a financial instrument that trades the probability of a rate hike (or cut) at each monetary policy meeting of BOJ.

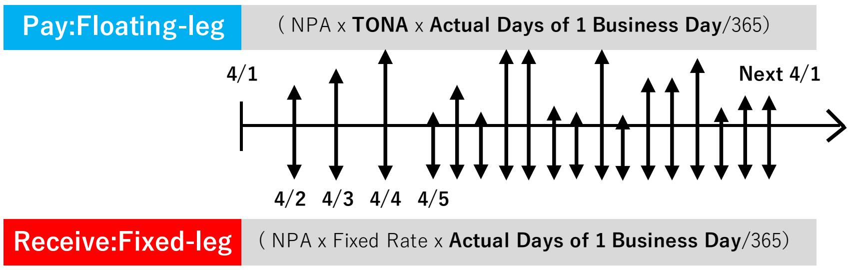
In December 2016, "TONA" was specified as Japanese Yen RFR, and is attracting attention as an alternative reference rate benchmark to Yen LIBOR. And Yen OIS, a short-term financial instrument created in 1997, is now called "TONA swap" and has been recognized into a financial instrument trading up to 40 years in duration.

[2] There are two types of fixed interest payment frequency for TONA swap: once a year (: PA) and twice a year (: SA).

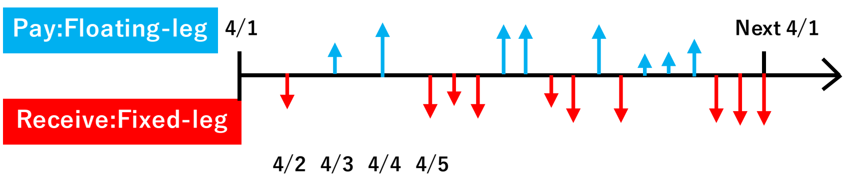
3. Timing of Fixing and Settlement in TONA Swap

TONA is calculated on the basis of overnight trading. And it could be settled for difference every business day using the "final result" of TONA announced by BOJ.

As an example, here is a cash flow figure of a standard TONA swap as receiving fixed-rate interest against paying floating-rate interest over 1 year, assuming the settlement every business day (Downward: Receiving, Upward: Paying, Horizontal Axis: Flow of Time).



The only thing to be settled is the difference between the fixed-rate interest and floating-rate interest, as shown in the figure below, but the administration work of settling every business day for a trading period of more than 1 year is troublesome and impractical.

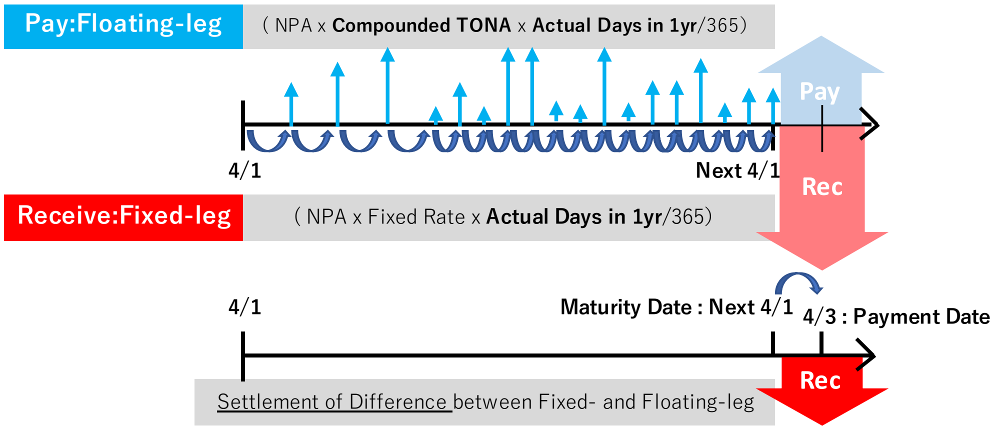


Settlement of Difference between Fixed- and Floating-leg on every business day

Therefore, the "compounding" method is used to reinvest the total principal and interest each business day in order to minimize the administration work while maintaining the same effect as settling each business day. When actually using the final result of TONA as the reference interest rate, TONA for up to 12 months will be compounded.

The figure below shows the cash flow in reality of a standard TONA swap transaction for receiving fixed-rate interest against paying floating-rate interest over 1 year. Normally, the number of payments for over 1 year is once a year, and only the difference between the fixed-rate interest and the compounded TONA interest for 12-month is settled.

It is important to note that the settlement date (: **Payment Date**) will be April 3 of the following year, 2 business days after the end of the swap duration (: **Maturity Date**). The main reason is that the final result of the last TONA (March 31-April 1 of the following year) for this swap will be announced at around 10:00 am on April 1, and overseas financial institutions with time differences need some time to ensure that the settlement operations are done properly.



4. "Compound Interest" and "Simple Interest”

You know that there are two types of interest calculations for financial instruments: Simple interest and Compound interest.

In simple interest calculation, the interest amount is the same because the interest is paid on the original principal, but in compound interest calculation, the principal continues to be invested along with the interest, so if the interest rate is positive, the interest amount will increase more and more.

Period	: 3 years
Int Rate	: 3%
Principal	: ¥1mio

Now, let's consider the case where the principal of 1,000,000 yen is invested in "Simple Interest" and "Compound Interest" with a period of 3 years and an interest rate of 3%.

The table below shows the results of a "Simple Interest" investment. I don't think a detailed explanation is necessary. If the interest rate is 3% per annum on the principal of 1,000,000 yen, the interest will be 30,000 yen every year for a total of 90,000 yen for 3 years.

Simple	Principal	Int Rate	Interest	Total	Calculation Formula
1st Year	1,000,000	3%	30,000	1,030,000	=1,000,000+1,000,000x3%
2nd Year	1,000,000	3%	30,000	1,060,000	=1,000,000+1,000,000x3%x2
3rd Year	1,000,000	3%	30,000	1,090,000	=1,000,000+1,000,000x3%x3

The 1st year is the same as in the case of Simple Interest, where the principal of 1,000,000 yen is invested at 3% interest rate for 1 year, so the total amount will be 1,030,000 yen after 1 year. The difference between Simple Interest and Compound Interest appears in the way of operation after the 2nd year.

Compounded	Principal	Int Rate	Interest	Total	Calculation Formula
1st Year	1,000,000	3%	30,000	1,030,000	=1,000,000+1,000,000x3%
2nd Year	1,030,000	3%	30,900	1,060,900	=1,030,000+1,030,000x3%
3rd Year	1,060,900	3%	31,827	1,092,727	=1,060,900+1,060,900x3%

In other words, the total principal and interest of 1,030,000 yen, which includes 30,000 yen of interest in the 1st year, will be invested for 1 year at 3% interest rate, so after 2 years it will be 1,069,900 yen. This 900 yen is the interest on the interest (30,000 yen). In the 3rd year, the total principal and interest of the 2nd year, 1,069,900 yen, will be invested for 1 year at 3% interest rate, so the amount will be 1,092,727 yen after 3 years. 1,827 yen of interest in the 3rd year is the interest on the interest (60,900 yen). In this way, "Compound Interest" invests the principal along with the interest.

The formula for calculating the total principal and interest in the case of Simple Interest and Compound Interest can be expressed as follows. In the case of Simple Interest, the interest portion is multiplied by the number of years, while in the case of Compound Interest, (1+interest) is multiplied by the number of years.[3]

Total principal and interest (: **Simple**) = principal x (1 + interest rate x number of years)
Total principal and interest (: **Compounded**) = principal x (1 + interest rate) $^{\text{number of years}}$

5. Basic concept of calculating "Compound TONA"

We will apply the basic concept of "compound interest" explained before to "Compounded TONA" calculation. The difference between the two is in the calculation of <u>compound interest every year</u> and <u>compound interest every business day</u>. So, if we change the number of years in the formula to the number of days in the period, and also modify it to take into account that TONA is for one business day, we get the following.

Total principal and interest (: Compounded) = principal x (1 + TONA x 1/365) $^{\text{number of days}}$

Once we know the total principal and interest, we can calculate Compounded TONA. This is because the total principal and interest consists of the original principal and the interest on the original principal (: Compounded TONA). (The following formula is based on the assumption that original principal = 1)

Total principal and interest (: Compounded) = 1 + (Compounded TONA x actual days/365)
Transforming the above, we get the following formula.
Compounded TONA = (Total principal and interest – 1) x 365/actual days

[3] Total principal and interest amount (: compounded) = principal x (1 + interest rate) $^{\text{number of years}}$ is a variation of **FV = PVx(1+R)^n** (FV: Future Value, PV: Present Value, R: Interest Rate, n: number of years).

Period	: 3 days
Int Rate	: TONA (pa)
Princilal	: 1

Now, let's take an example of compounding principal 1 at interest rate of TONA (pa) for 3 days.

If principal 1 is invested for 1 day at TONA, the interest is 1x(TONA x1/365), and the total principal and interest is 1+1x(TONA x1/365).

Compounded	Principal	Int Rate	Interest	Total	Calculation Formula
1st Day	1	TONA	1x(TONAx1/365)	(a)	=1+1x(TONAx1/365)
2nd Day	(a)	TONA	(a)x(TONAx1/365)	(A)	=(a)+(a)x(TONAx1/365)
3rd Day	(A)	TONA	(A)x(TONAx1/365)	(B)	=(A)+(A)x(TONAx1/365)

On the 2nd day, the total principal and interest (: a), which is the original principal 1 plus the interest on the 1st day, is invested at TONA, and the total principal and interest (: A) is (a)+(a)x(TONA x 1/365).

On the 3rd day, the total principal and interest (: B), if the total principal and interest (: A) was invested on the 2nd day at TONA, is (A)+(A)x(TONA x 1/365).

For example, if TONA is at the same interest rate for the entire year and is compounded for 365 days including weekends, the following formula can be used to calculate the compound TONA for 1 year.

Total principal and interest (: Compounded) = principal x $(1+\text{TONA}x1/365)^{365}$

However, in reality, since TONA changes every business day, and since there is a rule in Compounded TONA that "compounding is not performed on weekends and holidays," this formula cannot be adopted for Compounded TONA. Therefore, as shown in the following example, we got to calculate the total principal and interest for each business day and multiply by them.

6. Example of calculating “Compounded TONA”

In the previous chapter, we have learned that if we know the total principal and interest, we can calculate Compounded TONA.

The order of calculating "Compounded TONA" is as follows.

(1) Calculating Total Principal and Interest for each Business Day
(2) Multiplying by Total Principal and Interest for each Business Day.
(3) Deriving “Compounded TONA” from Total Principal and Interest

Perod	: 2w (: 14days)
Start Date	: 2021/9/13
Maturity Date	: 2021/9/27
TONA	: 3%
NPA	: 1

As an example, we will look at the concept of calculating the "2-week's Compounded TONA" for a trade of 2-week TONA swap.

It was assumed that the period is from September 13, 2021 to September 27, 2021, the principal is 1, and the final result of TONA

September			2021			
Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13 Start	14	15	16	17	18
19	20 Holiday	21	22	23 Holiday	24	25
26	27 End	28	29	30		

every business day is 3%.

Look at the calendar on the left. Within the period covered, there are 2 public holidays in addition to the regular weekends. Please be careful on how to calculate the total principal and interest in this case.

(1) Calculating Total Principal and Interest for each Business Day

In the table below, "Business Day" is the date and day of each business day from September 13, 2021 (: Start Date) to September 27, 2021 (: End Date), "TONA" is the final result of TONA (3%) for each business day from September 13 to 24 (: the business day prior to September 27), and "Days/Period" is the actual number of days in each business day and its duration.

The total principal and interest (: a~h) for each business day from Start Date (9/13) to End Date (9/27), if TONA (3%) is used, is as follows.

Business Day		TONA	Days	Period	Total Principal and Interest	Calculation Fomura
2021/9/13	Mon	3.0%	1	9/13 ～ 14	1.00008219178082	(a) =1+(3%x1/365)
2021/9/14	Tue	3.0%	1	9/14 ～ 15	1.00008219178082	(b) =1+(3%x1/365)
2021/9/15	Wed	3.0%	1	9/15 ～ 16	1.00008219178082	(c) =1+(3%x1/365)
2021/9/16	Thu	3.0%	1	9/16 ～ 17	1.00008219178082	(d) =1+(3%x1/365)
2021/9/17	Fri	3.0%	4	9/17 ～ 21	1.00032876712329	(e) =1+(3%x4/365)
2021/9/21	Tue	3.0%	1	9/21 ～ 22	1.00008219178082	(f) =1+(3%x1/365)
2021/9/22	Wed	3.0%	2	9/22 ～ 24	1.00016438356164	(g) =1+(3%x2/365)
2021/9/24	Fri	3.0%	3	9/24 ～ 27	1.00024657534247	(h) =1+(3%x3/365)
2021/9/27	Mon					

The formula to calculate the total principal and interest (: a) when invested at 3% of TONA on principal 1 for 1 day on 9/13-14 is 1+(3%x1/365); the formula to calculate the total principal and interest (: b) when invested at 3% of TONA on principal 1 for 1 day on 9/14-15 is 1+(3%x1/365). In the same way, the formula to calculate the total principal and interest (: c~h) when invested at 3% of TONA for principal 1 for each business day from 9/15 to 9/16, 9/16 to 9/17, 9/17 to 9/21, 9/21 to 9/22, 9/22 to 9/24, and 9/24 to 9/27 is 1+(3%x actual days/365).

(2) Multiplying by Total Principal and Interest for each Business Day.

Next, calculate the total principal and interest (: G) when compounded at TONA (3%) for 2 weeks from September 13 to 27.

The 2-week period of 9/13-27 consists of 8 parts: 9/13-14, 9/14-15, 9/15-9/16, 9/16-9/17, 9/17-9/21, 9/21-9/22, 9/22-9/24, and 9/24-9/27. The total amount of principal and interest for each of them (:

a~h) has already been calculated, and multiplying by them (: a~h) become the total principal and interest for 9/13 to 9/27.

Total principal and interest for 9/13-27 (: G)= (a)x(b)x(c)x(d)x(e)x(f)x(g)x(h)

Period	Totan Principal and Interest	Calculation Fomura	Compounded TONA	Calculation Fomura
9/13 ～ 15	1.00016439031713	(A) =(a)x(b)	3.00012%	=((A)-1)x365/2
9/13 ～ 16	1.00024659560949	(B) =(a)x(b)x(c)	3.00025%	=((B)-1)x365/3
9/13 ～ 17	1.00032880765844	(C) =(a)x(b)x(c)x(d)	3.00037%	=((C)-1)x365/4
9/13 ～ 21	1.00065768288288	(D) =(a)x(b)x(c)x(d)x(e)	3.00068%	=((D)-1)x365/8
9/13 ～ 22	1.00073992871983	(E) =(a)x(b)x(c)x(d)x(e)x(f)	3.00082%	=((E)-1)x365/9
9/13 ～ 24	1.00090443391359	(F) =(a)x(b)x(c)x(d)x(e)x(f)x(g)	3.00108%	=((F)-1)x365/11
9/13 ～ 27	1.00115123226716	(G) =(a)x(b)x(c)x(d)x(e)x(f)x(g)x(h)	3.00143%	=((G)-1)x365/14

(3) Deriving “Compounded TONA” from Total principal and interest

Assuming the notional principal amount is 1, the following relationship is established.

Total principal and interest (: Compounded) = 1 + (Compounded TONA x actual number of days/365)

Compounded TONA = (Total principal and interest –1) x365/ actual number of days

Since the total principal and interest for 9/13 - 9/27 (: G) assumes a notional principal amount of 1, Compounded TONA for 9/13 - 9/27 can be calculated as follows.

Compounded TONA for 9/13～27 = ((G) -1) x 365/14

= (1.00115123226716 - 1) x 365/14

= 3.00143%

By the way, there are 3 points to keep in mind when calculating Compounded TONA.

(1) TONA for Saturdays, Sundays, and national holidays is not compounded, but is applied as is. Therefore, in this case, the number of times compounded in 2 weeks is 8 times for 14 days.

(2) For the calculating the total principal and interest (as interim result), figures to be truncated to the 14th decimal place.

(3) Regarding the treatment of fractions in "Compounded TONA", 5 decimal places (: 3.00143%) will be used after rounding to the nearest 6th decimal place.

The end.

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